Solving Systems of Equations Using Matrices

Recall multiplication of matrices:

$$\begin{bmatrix} 3 & 1 \\ 2 & -5 \end{bmatrix} \times \begin{bmatrix} 7 \\ 4 \end{bmatrix} = \begin{bmatrix} (3)(7)+(1)(4) \\ (2)(7)+(-5)(4) \end{bmatrix} = \begin{bmatrix} 25 \\ -6 \end{bmatrix}$$

2x2 (r x c) ↑ order

(2x2) (2x1) ← match

new matrix

2x1 ← rows, column

2x1

Think about arithmetic multiplication:

$3 \times 4 = 12$
$4 \times 3 = 12$

} same answer, the order does not matter (arithmetic multiplication is commutative)

Now think about matrix multiplication.

$A \cdot B = AB$ ← this has a solution (2x3 matrix)

2x4 4x3 → 2x3

same

$B \cdot A$ = no solution (the columns from B do not match the rows from A)

4x3 2x4

do not match

MATRIX MULTIPLICATION IS NOT COMMUTATIVE... the order matters!!!

Solving equations:

$\frac{3x}{3} = \frac{12}{3}$ another way to write $\Rightarrow$ $x = 4$

$3x = 12$

$3\left(\frac{1}{3}\right)x = \frac{1}{3}(12)$

$1x = 4$

or $\boxed{x = 4}$

Now think about a matrix equation:

$AX = B$ (A, X and B are matrices)

$AA^{-1}X = A^{-1}B$

A^{-1}: inverse of matrix A (A must be a square matrix: 2x2, 3x3, 6x6 etc)

$IX = A^{-1}B$

Identity matrix (like 1) $\leftarrow$ I multiplied by

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = A$ $IC = C$

$IB = B$

$IX = X$

$\boxed{X = A^{-1}B}$

To summarize:

$AX = B$

$AA^{-1}X = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

Example

$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} X = \begin{bmatrix} -18 & -20 \\ 8 & 9 \end{bmatrix}$

$(AX = B)$

$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= I$

$X = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}^{-1}\begin{bmatrix} -18 & -20 \\ 8 & 9 \end{bmatrix}$

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[A]^-1[B]
   [[-146  -163]
    [ 60    67 ]]
```

$X = \begin{bmatrix} -146 & -163 \\ 60 & 67 \end{bmatrix}$

Example:

$\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} X + \begin{bmatrix} 2 & 4 \\ -3 & 10 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix}$

A B C

$AX + B = C$

$AX = C - B$

$X = A^{-1}(C - B)$

$X = \begin{bmatrix} \frac{17}{7} & -\frac{5}{7} \\ -\frac{5}{7} & \frac{6}{7} \end{bmatrix}$

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[A]^-1([C]-[B])
[[2.428571429 ...
 [-.7142857143 ...
Ans▸Frac
   [[17/7  -5/7]
    [-5/7  6/7 ]]
```

$3x + 6 = 10$